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CONVERSION TABLES FOR PID CONTROLLERS

PŘEVODNÍ TABULKY PRO PID REGULÁTORY

Abstract

The article deals with the mutual conversion of the controller adjustable parameters for six different transfer function forms of the PID controllers. These considered forms are the most frequent used in the technical experience.

Abstrakt

Článek pojednává o vzájemném převodu mezi stavitelnými parametry pro šest různých tvarů přenosů PID regulátorů. Uvažované tvary jsou v praxi používány nejčastěji.

1 INTRODUCTION

PID controllers belong among the most frequently used controllers in practice [Åström & Hägglund, 1995, 2006; O´Dwyer, 2003, 2006]. It is given their simple tuning and ability to assure relatively high-quality control performance.

Some problems arise when the standard PID controllers contain filtration, which can be placed in the control error or in the derivative term. Because the PID controller transfer functions have the different forms [Åström & Hägglund, 1995, 2006; O´Dwyer, 2003, 2006; Šulc, Vítečková, 2003], the conversion of the controller adjustable parameters can be troublesome, therefore it is supposed, that the filtration has neglectable influence on their values [Åström & Hägglund, 1995, 2006; O´Dwyer, 2003, 2006]. For greater values of the filtration time constant T_f this assumption isn't good and it can lead to incorrect results. Also from comparing different approaches and methods for PID controller tuning the resulting conclusions can be mistaken.

With respect to up-to-now in special literature, there does not exist summary relations for the mutual conversion of the PID controller adjustable parameters. The article brings the conversion formulas in the transparent tables.

2 TRANSFER FUNCTION FORMS OF PID CONTROLLERS

There are considered the following transfer function forms of the standard PID controllers [Åström & Hägglund, 1995, 2006; O´Dwyer, 2003, 2006; Šulc, Vítečková, 2003]:

$$G_R(s) = k_P\left(1+\frac{1}{T_I s}+T_D s\right)\frac{1}{T_f s+1} \qquad (1)$$

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$$G_R(s) = k'_P\left(1+\frac{1}{T'_I s}\right)(1+T'_D s)\frac{1}{T_f s+1} \tag{2}$$

$$G_R(s) = k''_P\left(1+\frac{1}{T''_I s}+\frac{T''_D s}{T_f s+1}\right) \tag{3}$$

$$G_R(s) = k'''_P\left(1+\frac{1}{T'''_I s}\right)\left(1+\frac{T'''_D s}{T_f s+1}\right) \tag{4}$$

$$G_R(s) = \left(r_0+\frac{r_{-1}}{s}+r_1 s\right)\frac{1}{T_f s+1} \tag{5}$$

$$G_R(s) = r'_0+\frac{r'_{-1}}{s}+\frac{r'_1 s}{T_f s+1} \tag{6}$$

Tab. 1

SOURCE TRANSFER FUNCTION	$k_P\left(1+\frac{1}{T_I s}+T_D s\right)\frac{1}{T_f s+1}$		
CONVERTED TRANSFER FUNCTION	ADJUSTABLE PARAMETERS		NOTE
$k'_P\left(1+\frac{1}{T'_I s}\right)(1+T'_D s)\frac{1}{T_f s+1}$	k'_P	$k_P\beta$	$\beta=\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{T_D}{T_I}}$ $0\le\frac{T_D}{T_I}\le\frac{1}{4}$
	T'_I	$T_I\beta$	
	T'_D	$T_D\frac{1}{\beta}$	
$k''_P\left(1+\frac{1}{T''_I s}+\frac{T''_D s}{T_f s+1}\right)$	k''_P	$k_P\gamma_1$	$\gamma_1=1-\frac{T_f}{T_I}$
	T''_I	$T_I\gamma_1$	
	T''_D	$T_D\frac{1}{\gamma_1}-T_f$	
$k'''_P\left(1+\frac{1}{T'''_I s}\right)\left(1+\frac{T'''_D s}{T_f s+1}\right)$	k'''_P	$k_P\beta$	$\beta=\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{T_D}{T_I}}$ $0\le\frac{T_D}{T_I}\le\frac{1}{4}$
	T'''_I	$T_I\beta$	
	T'''_D	$T_D\frac{1}{\beta}-T_f$	
$\left(r_0+\frac{r_{-1}}{s}+r_1 s\right)\frac{1}{T_f s+1}$	r_0	k_P	
	r_{-1}	$\frac{k_P}{T_I}$	
	r_1	$k_P T_D$	
$r'_0+\frac{r'_{-1}}{s}+\frac{r'_1 s}{T_f s+1}$	r'_0	$k_P\gamma_1$	$\gamma_1=1-\frac{T_f}{T_I}$
	r'_{-1}	$\frac{k_P}{T_I}$	
	r'_1	$k_P(T_D-T_f\gamma_1)$	

where k_P, k'_P, k''_P, k'''_P are the controller gains; T_I, T'_I, T''_I, T'''_I – the integral times; T_D, T'_D, T''_D, T'''_D – the derivative times; r_0, r'_0 – the proportional term weights; r_{-1}, r'_{-1} – the integral term weights; r_1, r'_1 – the derivative term weights; T_f – the filtration time constant.

The relations for converting mutual adjustable parameters for the PID controllers (1) – (6) are given in Tab. 1 – 6. The authors' tendency was so as to the relations there have an unified form. From the tables, it is obvious, that some relations are trivial but some are relatively complex.

Tab. 2

SOURCE TRANSFER FUNCTION	$k'_P\left(1+\frac{1}{T'_I s}\right)\left(1+T'_D s\right)\frac{1}{T_f s+1}$		
CONVERTED TRANSFER FUNCTION	ADJUSTABLE PARAMETERS		NOTE
$k_P\left(1+\frac{1}{T_I s}+T_D s\right)\frac{1}{T_f s+1}$	k_P	$k'_P i$	$i=1+\frac{T'_D}{T'_I}$
	T_I	$T'_I i$	
	T_D	$T'_D \frac{1}{i}$	
$k''_P\left(1+\frac{1}{T''_I s}+\frac{T''_D s}{T_f s+1}\right)$	k''_P	$k'_P i_1$	$i_1=1+\frac{T'_D-T_f}{T'_I}$
	T''_I	$T'_I i_1$	
	T''_D	$T'_D \frac{1}{i_1}-T_f$	
$k'''_P\left(1+\frac{1}{T'''_I s}\right)\left(1+\frac{T'''_D s}{T_f s+1}\right)$	k'''_P	k'_P	
	T'''_I	T'_I	
	T'''_D	T'_D-T_f	
$\left(r_0+\frac{r_{-1}}{s}+r_1 s\right)\frac{1}{T_f s+1}$	r_0	$k'_P i$	$i=1+\frac{T'_D}{T'_I}$
	r_{-1}	$\frac{k'_P}{T'_I}$	
	r_1	$k'_P T'_D$	
$r'_0+\frac{r'_{-1}}{s}+\frac{r'_1 s}{T_f s+1}$	r'_0	$k'_P i_1$	$i_1=1+\frac{T'_D-T_f}{T'_I}$
	r'_{-1}	$\frac{k'_P}{T'_I}$	
	r'_1	$k'_P(T'_D-T_f i_1)$	

Tab. 3

<table>
<tr><td>SOURCE TRANSFER FUNCTION</td><td colspan="3">$k''_P\left(1+\frac{1}{T''_I s}+\frac{T''_D s}{T_f s+1}\right)$</td></tr>
<tr><td>CONVERTED TRANSFER FUNCTION</td><td colspan="2">ADJUSTABLE PARAMETERS</td><td>NOTE</td></tr>
<tr><td rowspan="3">$k_P\left(1+\frac{1}{T_I s}+T_D s\right)\frac{1}{T_f s+1}$</td><td>$k_P$</td><td>$k''_P\gamma_2$</td><td rowspan="3">$\gamma_2=1+\frac{T_f}{T''_I}$</td></tr>
<tr><td>T_I</td><td>$T''_I\gamma_2$</td></tr>
<tr><td>T_D</td><td>$(T''_D+T_f)\frac{1}{\gamma_2}$</td></tr>
<tr><td rowspan="3">$k'_P\left(1+\frac{1}{T'_I s}\right)(1+T'_D s)\frac{1}{T_f s+1}$</td><td>$k'_P$</td><td>$k''_P\beta_2$</td><td rowspan="3">$\gamma_2=1+\frac{T_f}{T''_I}$
$\beta_2=\gamma_2\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{T''_D+T_f}{T''_I}\frac{1}{\gamma_2^2}}\right)$
$0\le\frac{T''_I(T''_D+T_f)}{(T''_I+T_f)^2}\le\frac{1}{4}$</td></tr>
<tr><td>T'_I</td><td>$T''_I\beta_2$</td></tr>
<tr><td>T'_D</td><td>$(T''_D+T_f)\frac{1}{\beta_2}$</td></tr>
<tr><td rowspan="3">$k'''_P\left(1+\frac{1}{T'''_I s}\right)\left(1+\frac{T'''_D s}{T_f s+1}\right)$</td><td>$k'''_P$</td><td>$k''_P\beta_2$</td><td rowspan="3">$\gamma_2=1+\frac{T_f}{T''_I}$
$\beta_2=\gamma_2\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{T''_D+T_f}{T''_I}\frac{1}{\gamma_2^2}}\right)$
$0\le\frac{T''_I(T''_D+T_f)}{(T''_I+T_f)^2}\le\frac{1}{4}$</td></tr>
<tr><td>T'''_I</td><td>$T''_I\beta_2$</td></tr>
<tr><td>T'''_D</td><td>$(T''_D+T_f)\frac{1}{\beta_2}-T_f$</td></tr>
<tr><td rowspan="3">$\left(r_0+\frac{r_{-1}}{s}+r_1 s\right)\frac{1}{T_f s+1}$</td><td>$r_0$</td><td>$k''_P\gamma_2$</td><td rowspan="3">$\gamma_2=1+\frac{T_f}{T''_I}$</td></tr>
<tr><td>r_{-1}</td><td>$\frac{k''_P}{T''_I}$</td></tr>
<tr><td>r_1</td><td>$k''_P(T''_D+T_f)$</td></tr>
<tr><td rowspan="3">$r'_0+\frac{r'_{-1}}{s}+\frac{r'_1 s}{T_f s+1}$</td><td>$r'_0$</td><td>$k''_P$</td><td rowspan="3"></td></tr>
<tr><td>r'_{-1}</td><td>$\frac{k''_P}{T''_I}$</td></tr>
<tr><td>r'_1</td><td>$k''_P T''_D$</td></tr>
</table>

Tab. 4

SOURCE TRANSFER FUNCTION	$k_P'''\left(1+\frac{1}{T_I'''s}\right)\left(1+\frac{T_D'''s}{T_f s+1}\right)$		
CONVERTED TRANSFER FUNCTION	ADJUSTABLE PARAMETERS		NOTE
$k_P\left(1+\frac{1}{T_I s}+T_D s\right)\frac{1}{T_f s+1}$	k_P	$k_P''' i_3$	$i_3=1+\frac{T_D'''+T_f}{T_I'''}$
	T_I	$T_I''' i_3$	
	T_D	$(T_D'''+T_f)\frac{1}{i_3}$	
$k_P'\left(1+\frac{1}{T_I's}\right)(1+T_D's)\frac{1}{T_f s+1}$	k_P'	k_P'''	
	T_I'	T_I'''	
	T_D'	$T_D'''+T_f$	
$k_P''\left(1+\frac{1}{T_I''s}+\frac{T_D''s}{T_f s+1}\right)$	k_P''	$k_P''' i_2$	$i_2=1+\frac{T_D'''}{T_I'''}$
	T_I''	$T_I''' i_2$	
	T_D''	$(T_D'''+T_f)\frac{1}{i_2}-T_f$	
$\left(r_0+\frac{r_{-1}}{s}+r_1 s\right)\frac{1}{T_f s+1}$	r_0	$k_P''' i_3$	$i_3=1+\frac{T_D'''+T_f}{T_I'''}$
	r_{-1}	$\frac{k_P'''}{T_I'''}$	
	r_1	$k_P'''(T_D'''+T_f)$	
$r_0'+\frac{r_{-1}'}{s}+\frac{r_1's}{T_f s+1}$	r_0'	$k_P'''\left(1+\frac{T_D'''}{T_I'''}\right)$	
	r_{-1}'	$\frac{k_P'''}{T_I'''}$	
	r_1'	$k_P''' T_D'''\left(1-\frac{T_f}{T_I'''}\right)$	

Tab. 5

SOURCE TRANSFER FUNCTION	$\left(r_0+\frac{r_{-1}}{s}+r_1 s\right)\frac{1}{T_f s+1}$		
CONVERTED TRANSFER FUNCTION	ADJUSTABLE PARAMETERS		NOTE
$k_P\left(1+\frac{1}{T_I s}+T_D s\right)\frac{1}{T_f s+1}$	k_P	r_0	
	T_I	$\frac{r_0}{r_{-1}}$	
	T_D	$\frac{r_1}{r_0}$	
$k'_P\left(1+\frac{1}{T'_I s}\right)\left(1+T'_D s\right)\frac{1}{T_f s+1}$	k'_P	$r_0\beta_1$	$\beta_1=\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{r_1 r_{-1}}{r_0^2}}$
	T'_I	$\frac{r_0}{r_{-1}}\beta_1$	$0\le\frac{r_1 r_{-1}}{r_0^2}\le\frac{1}{4}$
	T'_D	$\frac{r_1}{r_0}\frac{1}{\beta_1}$	
$k''_P\left(1+\frac{1}{T''_I s}+\frac{T''_D s}{T_f s+1}\right)$	k''_P	$r_0\gamma_3$	$\gamma_3=1-\frac{r_{-1}}{r_0}T_f$
	T''_I	$\frac{r_0}{r_{-1}}\gamma_3$	$0\le\frac{r_{-1}}{r_0}T_f<1$
	T''_D	$\frac{r_1}{r_0}\frac{1}{\gamma_3}-T_f$	
$k'''_P\left(1+\frac{1}{T'''_I s}\right)\left(1+\frac{T'''_D s}{T_f s+1}\right)$	k'''_P	$r_0\beta_1$	$\beta_1=\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{r_1 r_{-1}}{r_0^2}}$
	T'''_I	$\frac{r_0}{r_{-1}}\beta_1$	$0\le\frac{r_1 r_{-1}}{r_0^2}\le\frac{1}{4}$
	T'''_D	$\frac{r_1}{r_0}\frac{1}{\beta_1}-T_f$	
$r'_0+\frac{r'_{-1}}{s}+\frac{r'_1 s}{T_f s+1}$	r'_0	$r_0\gamma_3$	$\gamma_3=1-\frac{r_{-1}}{r_0}T_f$
	r'_{-1}	r_{-1}	
	r'_1	$r_1-r_0 T_f\gamma_3$	$0\le\frac{r_{-1}}{r_0}T_f<1$

Tab. 6

SOURCE TRANSFER FUNCTION	$r_0' + \frac{r_{-1}'}{s} + \frac{r_1' s}{T_f s + 1}$		
CONVERTED TRANSFER FUNCTION	ADJUSTABLE PARAMETERS		NOTE
$k_P\left(1 + \frac{1}{T_I s} + T_D s\right)\frac{1}{T_f s + 1}$	k_P	$r_0'\gamma_4$	$\gamma_4 = 1 + \frac{r_{-1}'}{r_0'}T_f$
	T_I	$\frac{r_0'}{r_{-1}'}\gamma_4$	
	T_D	$\left(\frac{r_1'}{r_0'} + T_f\right)\frac{1}{\gamma_4}$	
$k_P'\left(1 + \frac{1}{T_I' s}\right)(1 + T_D' s)\frac{1}{T_f s + 1}$	k_P'	$r_0'\beta_3$	$\gamma_4 = 1 + \frac{r_{-1}'}{r_0'}T_f$ $\beta_3 = \gamma_4\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{r_1' r_{-1}'}{r_0'^2}\left(1 + \frac{r_0'}{r_1'}T_f\right)\frac{1}{\gamma_4^2}}\right)$ $0 \le \frac{r_{-1}'(r_1' + r_0' T_f)}{(r_0' + r_{-1}' T_f)^2} \le \frac{1}{4}$
	T_I'	$\frac{r_0'}{r_{-1}'}\beta_3$	
	T_D'	$\left(\frac{r_1'}{r_0'} + T_f\right)\frac{1}{\beta_3}$	
$k_P''\left(1 + \frac{1}{T_I'' s} + \frac{T_D'' s}{T_f s + 1}\right)$	k_P''	r_0'	
	T_I''	$\frac{r_0'}{r_{-1}'}$	
	T_D''	$\frac{r_1'}{r_0'}$	
$k_P'''\left(1 + \frac{1}{T_I''' s}\right)\left(1 + \frac{T_D''' s}{T_f s + 1}\right)$	k_P'''	$r_0'\beta_3$	$\gamma_4 = 1 + \frac{r_{-1}'}{r_0'}T_f$ $\beta_3 = \gamma_4\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{r_1' r_{-1}'}{r_0'^2}\left(1 + \frac{r_0'}{r_1'}T_f\right)\frac{1}{\gamma_4^2}}\right)$ $0 \le \frac{r_{-1}'(r_1' + r_0' T_f)}{(r_0' + r_{-1}' T_f)^2} \le \frac{1}{4}$
	T_I'''	$\frac{r_0'}{r_{-1}'}\beta_3$	
	T_D'''	$\left(\frac{r_1'}{r_0'} + T_f\right)\frac{1}{\beta_3} - T_f$	
$\left(r_0 + \frac{r_{-1}}{s} + r_1 s\right)\frac{1}{T_f s + 1}$	r_0	$r_0' + r_{-1}' T_f$	
	r_{-1}	r_{-1}'	
	r_1	$r_1' + r_0' T_f$	

3 CONCLUSIONS

In the article are given the conversion formulas in the form of the transparent tables. The formulas enable the very quick mutual conversion between six different forms of the PID transfer functions.

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